Compressible Laminar Boundary Layers on Sharp Cone at Incidence with Entropy Swallowing

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Theme

THE Navier-Stokes equations for steady motion of compressible viscous fluid flow past a body are highly nonlinear and elliptic and present formidable obstacles to classical analytical or numerical methods of solution except in special cases. By making the boundary-layer approximation, these equations are reduced to parabolic type and represent an initial-two-point-boundary-value problem which can be solved by a marching technique. In general, the resulting equations are solved numerically for specified boundary conditions at the wall and for prescribed external inviscid fluid properties at the edge of the boundary layer.

To satisfy the boundary conditions requirement of the boundary-layer equations, external inviscid properties are needed. The usual assumption for the external flow at the edge of the boundary layer is that it is of constant entropy corresponding to either the oblique shock entropy or stagnation streamline entropy. However, as the boundary layer thickens in the streamwise, and cross-flow directions in three-dimensional flow, the streamlines around the periphery of the cone are passing through a shock of varying strength and, therefore, posses different entropies. These streamlines of variable entropy are entrained into the boundary layer and the isentropic conditions, especially for a body at an angle of attack, becomes invalid. This phenomenon known as entropy swallowing will occur on hypersonic lifting vehicles and on space-shuttle-type vehicles at angle of attack. It is of practical importance to know the effects of variable entropy on the magnitude of skin friction and heat transfer at the surface of a vehicle.

In the analysis presented herein, a new and simple method of accounting for the entropy swallowing effects is employed by specifying as edge conditions the actual inviscid flow conditions at a distance from the wall equal to the thickness of the boundary layer. Extensive computational results are obtained and compared with results by other methods and with experimental data. An implicit finite-difference technique[1,2] originally developed by McGowan and Davis[3] is employed for solving the three-dimensional compressible laminar boundary-layer equations over sharp right circular cones. The governing equations are modified by a similarity-type transformation for the normal coordinate and velocity and by another normal coordinate transformation. The resulting governing equations of Crocco-type form

$$\frac{\partial^2 W}{\partial \zeta^2} + \alpha_1 \frac{\partial W}{\partial \zeta} + \alpha_2 W + \alpha_3 + \alpha_4 \frac{\partial W}{\partial \xi} + \alpha_5 \frac{\partial W}{\partial \eta} = 0 \qquad (1)$$

are second-order parabolic partial differential equations in ζ and first-order equations in ξ and can be solved by marching technique. In this equation, W stands for any dependent variable and ξ,η,ζ and $u,v,$ and w coordinates and velocities along the cone generator, in the tangential and in normal directions, respectively. Introducing a second-order finite-difference scheme, the system of nonlinear second-order parabolic equations is reduced to algebraic equations having the tridiagonal form and is solved by an efficient algorithm. The α_i coefficients in Eq. (1) are functions of the inviscid flow properties and other dependent variables which are gussed initially and are subsequently updated during the iteration process.

The inviscid data used as boundary conditions for the boundary-layer equations was obtained from Jones.[4] It is critical for numerical stability of the boundary-layer solutions to use inviscid data which is consistent with the equations to be solved. To generate such a consistent set of the inviscid data for the case of entropy swallowing, the Bernoulli's equation fro compressible flow is used

$$v_e^2 + w_e^2 + u_e^2 + 2\left(\frac{pe}{\rho_e} \frac{\gamma}{\gamma - 1} - h_0 \right) = 0 \qquad (2)$$

The inviscid data to be used in these equations is interpolated at the edge of the boundary layer. However, since vorticity is present in the inviscid shock layer, the gradients of velocity and temperature are no longer zero at the edge and the boundary-layer thickness is not well defined. This difficulty is overcome by basing the defintion of the boundary-layer thickness on the invariance of the total enthalpy h_0.[5] Thus u_e may be computed and the edge of the boundary layer defined at the point where u approaches u_e. The variable entropy is introduced by expressing the density ρ in terms of the entropy parameter $k = p/\rho^\gamma$ which is evaluated at the edge.

Expressing the cross-flow velocity v_e from the ξ-momentum equation

$$v_e = \frac{\partial u_e}{\partial \eta} + \frac{\xi u_e}{v_e} \frac{\partial u_e}{\partial \xi} + \frac{\xi}{v_e \rho_e} \frac{\partial p_e}{\partial \xi} \qquad (3)$$

the derivative $\partial u_e / \partial \eta$ is introduced into the Bernoulli's equation which becomes

$$\left(\frac{\partial u_e}{\partial \eta} \right)^2 + u_e{}^2 + u_e{}^2 \left(\frac{\xi}{v_e} \frac{\partial u_e}{\partial \xi} \right)^2 + \left(\frac{\xi}{\rho_e v_e} \frac{\partial p_e}{\partial \xi} \right)^2$$
$$+ u_e \frac{\partial u_e}{\partial \eta} \left(\frac{2\xi}{v_e} \frac{\partial u_e}{\partial \xi} \right) + \frac{\partial u_e}{\partial \eta} \left(\frac{2}{\rho_e} \frac{\xi}{v_e} \frac{\partial p_e}{\partial \xi} \right)$$
$$+ 2u_e \left(\frac{\xi}{v_e} \frac{\partial u_e}{\partial \xi} \right) \left(\frac{\xi}{\rho_e v_e} \frac{\partial p_e}{\partial \xi} \right) + w_e{}^2$$
$$+ 2\left(\frac{2\gamma}{\gamma - 1} p_e{}^{\gamma - 1/\gamma} k^{1/\gamma} - h_0 \right) = F \qquad (4)$$

Replacing the drivative $\partial u_e / \partial \eta$ by a 5-point finite difference

$$\partial u_e / \partial \eta = 8(u_{j+1} - u_{j-1}) - (u_{j+2} - u_{j-2}) / 120\eta \qquad (5)$$

and applying Eq. (4) at discrete points around the body, a set

Received August 19, 1974; synoptic received September 23, 1974; revision received February 28, 1975. Full paper (NASA CR-112316) available from the National Technical Information Service, Springfield, Va., 22151 at the standard price (available upon request). This research was supported by NASA under Contract NAS1-11104.

Index categories: Boundary Layers and Convective Heat Transfer—Laminar; Supersonic and Hypersonic Flow.

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of simultaneous nonlinear algebraic equations written on u_e is obtained. Using inviscid data interpolated at a distance δ for p_e, k, w_e and u_e,v_e as initial values for fast iteration and evaluating the inviscid derivatives or using their values initially from previous stations, the system of equations is solved by the Newton-Raphson method for u_e by setting for F a sufficiently small value.

The cross-flow velocity v_e is improved from the initial value by applying ξ-momentum equation in an iterative scheme. The computed edge values are used to solve the boundary-layer equations for variable entropy. With the edge velocity u_e known, the edge of the boundary layer is defined at the point where $|u/u_e-1|\leqslant\epsilon$ and interpolation at this point is made for the next streamwise station.

Contents

The computed results include: skin friction, heat transfer, boundary-layer thickness, displacement thickness, Stanton number and the normal direction profiles of velocities, temperature, density, and shear parameter. The computations were performed at conditions: $M_\infty=8$; half-cone angle: $\phi_c=10°$, $\phi_c=25°$; $R_e=4.04\times10^4$ to 4.04×10^6; and angle of attack: $\alpha=8°$, $10°$, $12°$, $12.5°$.

The skin friction and heat transfer were normalized by values $C_{f\xi 0}$ and q_{wo} at zero angle of attack, respectively. Longitudinal and transverse skin friction and heat transfer distributions in the transverse direction for $\phi_c=10°$, and $\alpha=8°$, $10°$, and $12°$ and various values of the Reynolds number and for $\phi_c=25°$ and $\alpha=12.5°$ were computed for the constant entropy case (no swallowing) and for entropy swallowing. Heat transfer results from $\phi_c=10°$ are compared with the tests by Tracy.[6]

The longitudinal and transverse skin friction distributions for $\phi_c=10°$, $\alpha=8°$, and $R_e=4.04\times10^4$ are shown in Fig. 1. The Reynolds number is moderate and the boundary layer has a significant thickness. As a result of entropy swallowing the values of the skin friction are higher than those for constant entropy over almost the entire periphery: about 2% at the windward and 4% at the leeward region. A similar trend is displayed by the heat transfer in Fig. 2. Heat transfer distributions are in good agreement with the test values by Tracy[6] except toward the leeward region and windward region where the computed value is about 3% higher than the test results.

It can be concluded that the described method based on the Crocco-type form of transformation and implicit finite difference method is a fast and efficient method of obtaining the three-dimensional boundary-layer solutions with variable entropy effects for conical bodies and potentially for general three-dimensional flow with adequate inviscid data for which the boundary-layer theory remains valid. The effects of entropy swallowing result in higher values for heat transfer and skin friction than the values for constant entropy. This trend is consistent within the range of the ϕ_c, α, and R_e values used in these computations.

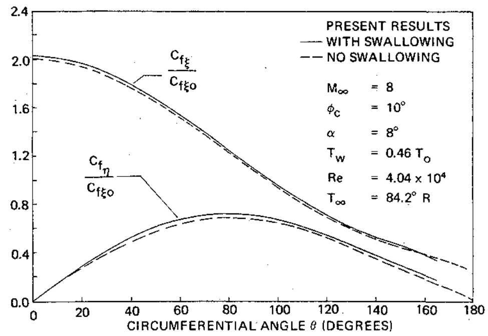

Fig. 1 Skin friction distribution on a sharp circular cone.

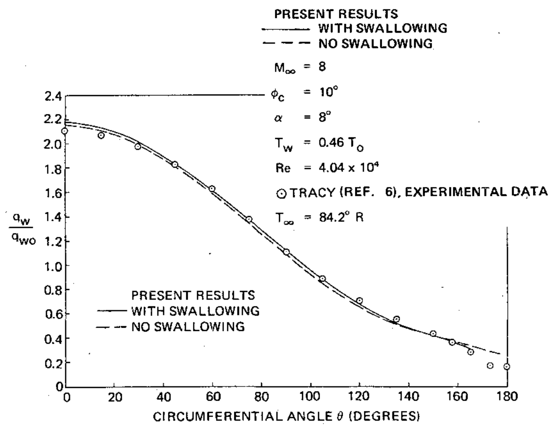

Fig. 2 Heat transfer distribution on a sharp circular cone.

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